The Optimization of Procurement Planning in Global Sourcing

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Global Sourcing: Practice of sourcing from the global market for goods or services under certain geopolitical constraints
Global Sourcing

Companies seek cooperation with suppliers in distant locations, where the costs of primary products or services are considerably low.
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- cost of manufacturing
- fluctuation of exchange rates
- availability of infrastructures
Global Sourcing

Companies seek cooperation with suppliers in distant locations, where the costs of primary products or services are considerably low.

- cost of manufacturing
- fluctuation of exchange rates
- availability of infrastructures
- LOGISTICS
- INVENTORY MANAGEMENT
- DISTANCE
Procurement Planning

We seek an optimal balance among the costs on setup, inventory holding, stockout penalty, etc.
Problem Definition

Lead time: $L$ periods

$x_i$: order quantity arriving at the beginning of period $i$. 

$N$ periods

$N$ periods

$N$ periods

$L-N$ periods
Problem Definition

order $x_i$

order $x_{i+1}$

order $x_{i+N-2}$

order $x_{i+N-1}$


$x_i$: order quantity arriving at the beginning of period $i$. 

$N$ periods

$L$-N periods
Objective Function

The objective of procurement planning is to specify the order quantities \( x_i, i = 1, 2, \cdots, N \), that produce the minimum total costs (including setup, inventory holding and backlogging costs), i.e.

\[
\text{Minimize} \quad \sum_{i=1}^{N} C_i
\]

where \( C_i \) denotes the cost incurred in period \( i = 1, 2, \cdots, N \),

\[
C_i = \begin{cases} 
K_i - L \delta(x_i) + h_i I_i, & \text{if } I_i \geq 0 \\
K_i - L \delta(x_i) + b_i (-I_i), & \text{if } I_i < 0 
\end{cases}
\]
Difficulty:
Large uncertainty and dramatic change of demand

- Demand can only be estimated by prediction from the past data in previous periods.
- Good predictions can bring a great help in the subsequent procurement planning.

Empirically, the forecast of demand in a certain period $i$ is gradually revised over time, which can be expressed as

$$
\epsilon_{i,i-1} \leq \epsilon_{i,i-2} \leq \cdots \leq \epsilon_{i,i-L} \leq \cdots \quad i = 1, 2, \cdots, N. \tag{3}
$$

$\epsilon_{i,k}$: Expected absolute forecast error when predicting demand of period $i$ at period $k$. 
Rolling Horizon Scheme

... i i i i i ... i i i i i ... n + n n n n ... n periods

1 2 3 4 n periods

1 1 2 3 n periods

n periods
Demand Forecasting

**Assumption:** at the observation time point (defined as the moment when the forecast is made), the forecasts of demand in the considered sub-horizon are ”good enough” so that *the demand in each period of the sub-horizon can be considered to follow some known distribution*, e.g., normal distribution.
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* The forecast of demand can be regarded as the expectation of the demand distribution.

* The variance of the distribution can be derived from the past forecast errors while considering the inherent characteristics of the procurement process, such as trend, cycle and seasonality.
Demand Forecasting

In sub-horizon $\eta_i$ (including periods $i$ to $i + n - 1$), the initial stock is given as

$$I_i^0 = I_{i-L}^0 + \sum_{j=i-L}^{i-1} (x_j - d_j) \quad (4)$$

where $I_{i-L}^0$ and $x_j$ are known at the observation time point (period $i - L$).

Suppose $d_j (j = i - L, \cdots, i - 1)$ follow some known distribution, then the distribution of $I_i^0$ can be estimated.
Heuristic Algorithm

Net inventory requirements in the sub-horizon:

<table>
<thead>
<tr>
<th>Period'</th>
<th>1</th>
<th>2</th>
<th>...</th>
<th>n-1</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expectation</td>
<td>$\mu_1$</td>
<td>$\mu_2$</td>
<td>...</td>
<td>$\mu_{n-1}$</td>
<td>$\mu_n$</td>
</tr>
<tr>
<td>Variance</td>
<td>$\sigma_1$</td>
<td>$\sigma_2$</td>
<td>...</td>
<td>$\sigma_{n-1}$</td>
<td>$\sigma_n$</td>
</tr>
</tbody>
</table>

- **Deterministic part**: $\mu_1, \ldots, \mu_n$
- **Stochastic part**: $\sigma_1, \ldots, \sigma_n$
Heuristic Algorithm

The core idea is to **decouple the deterministic part** \((\mu_1, \cdots, \mu_n)\) **and the stochastic part** \((\sigma_1, \cdots, \sigma_n)\) in the problem. The procurement planning for the sub-horizon can be determined in two stages:

1. determine the optimal procurement planning \(\{y_1, y_2, \cdots, y_n\}\) while considering the fixed demand \(\{\mu_1, \mu_2, \cdots, \mu_n\}\)

2. propose an adequate safety stock \(S\) to cope with the probable stockout caused by demand uncertainty.
Safety Stock

The principle is to find the adequate safety stock that \textit{minimizes the total expected cost} over the sub-horizon. The safety stock $S$ can be obtained by solving Equation:

$$\sum_{k=1}^{n} (h_k + b_k) \Phi\left( \frac{S + Y_k - M_k}{\sum_k} \right) = \sum_{k=1}^{n} b_k$$

(5)

where

$$\Phi\left( \frac{S + Y_k - M_k}{\sum_k} \right) = \int_{-\infty}^{S+Y_k} \frac{1}{\sqrt{2\pi} \cdot \sum_k} \cdot e^{-\frac{1}{2} \left( \frac{\Theta_k - M_k}{\sum_k} \right)^2} d\Theta_k$$

(6)
Conclusion

An adaptive optimization approach

Rolling-horizon planning:
Conclusion

An adaptive optimization approach

Rolling-horizon planning:

In each sub-horizon, the distribution of demand is estimated by prediction from the past data.
Conclusion

An adaptive optimization approach

Rolling-horizon planning:

- In each sub-horizon, the distribution of demand is estimated by prediction from the past data.
- Based on the predicted demand, a heuristic algorithm has been developed to solve the stochastic lot sizing problem in each sub-horizon.
Ex-post-facto Experiments

For each scenario of demand $\mathcal{D}^j = \{d_1^j, d_2^j, \cdots, d_N^j\}$, we have:

- $C^*_j$: the ex-post-facto minimum total cost by employing any classical deterministic dynamic lot-sizing algorithm
- $C_j$: the total cost by applying the proposed optimization approach
Ex-post-facto Experiments

For each scenario of demand $\mathcal{D}^j = \{d_1^j, d_2^j, \cdots, d_N^j\}$, we have:

- $C^*^j$ : the ex-post-facto minimum total cost by employing any classical deterministic dynamic lot-sizing algorithm
- $C^j$ : the total cost by applying the proposed optimization approach

\[ \frac{C^j}{C^*^j} : \text{performance ratio of the proposed approach applied on demand scenario } \mathcal{D}^j \]
Ex-post-facto Experiments

Regarding of the randomness of the problem, the final evaluation of the proposed approach should be a statistical result and not decided by a single demand scenario.

Letting $E^\kappa(\mathcal{I})$ denote the statistical mean among the objective value $\mathcal{I}$ of $\kappa$ randomly generated demand scenarios, we have two measures to evaluate the proposed approach, expressed respectively as

$$
\Gamma_1^\kappa = \frac{E^\kappa(C)}{E^\kappa(C^*)}, \quad \text{and} \quad \Gamma_2^\kappa = E^\kappa\left(\frac{C}{C^*}\right). \quad (7)
$$
Ex-post-facto Experiments

- Scenario $\mathcal{D}^j$
- Estimate initial stock $l_0$
- Predict the demand distribution in each period of sub-horizon
- Calculate net inventory requirements
- Determine $y_1, y_2, \ldots, y_n$
- Proposed Heuristic
- Determine $S$
- $x_i = y_1 + \max\{0, S\}$
- $i = i + 1$

- No
  - $i = N - n + 2$ ?
- Yes
  - Calculate $C^j$ and $C^{*j}$

Context
- Global Sourcing
- Procurement Planning

Problem
- Definition
- Objective

Methodology
- Difficulty
- Rolling Horizon
- Demand Forecasting
- Heuristic
- Conclusion

Results
- Ex-post-facto Experiments
- Theoretical Analysis
Ex-post-facto Experiments

An instance

To launch the experiments, a large amount of demand scenarios should be randomly generated at first.

Suppose the demand follows the form

\[ d_t = a T(t) + (b + c \times \text{randn}) \left[ d + \sin \left( \frac{2\pi}{e} \times (t + \frac{f}{4}) \right) \right] \] (8)

where \( t \) is the time period, \( T(t) \) is a polynomial function of \( t \), \( a, b, c, d, e, f \) are constant parameters, and \( \text{randn} \) is a random number that follows the standard normal distribution.
Ex-post-facto Experiments

An instance

Demand parameters: \( a = 0.001, T(t) = (t - 180)^2, b = 100, c = 50, d = 1, e = f = 180. \)

Planning horizon: \( N = 1000. \)

Sub-horizon: \( n = 6. \)

Cost: setup \( K = 500 \), inventory holding \( h = 1 \), backlogging \( b = 10 \).

Number of scenarios tested: 500.

Results: \( L = 1, \frac{C}{C^*} \in [2.03, 2.68], \Gamma_1^{500} = \Gamma_2^{500} = 2.34. \)

\( L = 50, \Gamma_1^{500} = \Gamma_2^{500} = 3.29. \)
Theoretical Analysis

- Efficiency of proposed heuristic: comparison with existing methods
Theoretical Analysis

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- Worst-case performance ratio:

$$\Omega = \sup \left( \frac{C}{C^*} \right)$$
Theoretical Analysis

• Efficiency of proposed heuristic: comparison with existing methods ✓

• Worst-case performance ratio:

\[ \Omega = \sup \left( \frac{C}{C^*} \right) \]
Theoretical Analysis

- Efficiency of proposed heuristic: comparison with existing methods ✓
- Worst-case performance ratio:
  \[ \Omega = \sup \left( \frac{C}{C^*} \right) \]
- Expected long-run performance ratio:
  \[ \Gamma^\infty = E^\infty \left( \frac{C}{C^*} \right) \]
Thank you!