Interdependent systems

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Objectives of this lesson

• What is the relevance of (inter-)dependencies?
• How to model these phenomena?
• Which are consequences if we neglect to capture them?
Dependency vs. interdependency

**Dependency**: is the capability of an infrastructure to influence the state of another infrastructure. It is a *unidirectional* relationship.

**Interdependency**: is a *bidirectional* relationship between two infrastructures through which the state of each infrastructure is influenced or is correlated to the state of the other.

Notice that in literature, with an abuse of notation, the term “Interdependency” is used with a broad meaning absorbing in part the “dependency” meaning.
First and higher order dependency

- **First order** dependency A → B
- **Second order** dependency A → C → B

The concept can be easily generalized to the \text{n-th order dependency}

When the sequence of influences creates a \text{loop}, A → C → B → A then ALL the involved infrastructures are inter-dependent. Any event is \textit{exacerbated}.

In the presence of loops, there is no more a \textit{tree} structure (i.e. there is a root and the consequences go only downstairs) but a \textit{graph} structure (the consequences have no more a preferential direction)
Dimensions for describing infrastructure interdependencies.

Dimensions for describing infrastructure interdependencies.

**Physical Interd.**: if the operations of one infrastructure depends on the physical output(s) of the other.

**Cyber Interd.**: if its state depends on information transmitted via cyberspace.

**Geographical Interd.**: when elements are in close spatial proximity.

**Logical Interd.**: any other causes (e.g. regulamentary)

**Sociologic Interd.**: when coupling effects are mediated by (irrational) human behaviors

Interdependency Metrics
How to measure dependency

A cornerstone question is how to measure the degree of (inter)dependency existing among any two infrastructure in order to qualify normal and pathological situation.

A general approach is those to evaluate the degree of dependency on a relative base, i.e. how much are amplified the negative consequences.

\[ I = \frac{f(\text{coupled}) - f(\text{atomic})}{f(\text{atomic})} \]
Dependency Measurement

The dependency index is the ratio between the relative increments of the inoperability in the depended infrastructure with respect to those experienced in the source infrastructure:

\[
\delta_{A,B}^{t} \left( \Delta x_{B}^{0} \right) = \frac{\int_{t_{0}}^{t_{0}+T} \Delta x_{A}(\tau) d\tau - \int_{t_{0}}^{t_{0}+T} x_{A}(\tau) d\tau}{\int_{t_{0}}^{t_{0}+T} \Delta x_{B}(\tau) d\tau - \int_{t_{0}}^{t_{0}+T} x_{B}(\tau) d\tau}
\]

Variation in the «inoperability» of A due the event occurred in B

Effect on B
Inter-Dependency Measurement

Internal interdependency index \( \delta^I_{A,B} \) is the ratio between the injected augment of inoperability \( \Delta x^o_B \) and effective increment in the level of inoperability in the same infrastructure, i.e., \( \Delta x_B(t) \).

External interdependency index \( \delta^E_{A,B} \) is the ratio between the injected augment of inoperability in infrastructure B (i.e., \( \Delta x^o_B \)) and increment in the level of inoperability experienced by infrastructure A (i.e., \( \Delta x_A(t) \)).
Interdependences modelling
Input-output Models

Infrastructures are modeled as black boxes

The emphasis is on interaction (input and output)

- Which inputs are needed?
- What is the effect of a lack of resources?
Input-Output Inoperability Model

- Based on the economic equilibrium theory of W. Leontief
- Each infrastructure has an **inoperability** q (% of malfunctioning)
- The model considers constant external perturbations and analyzes the domino effects


IIM - Example

Leontief Matrix. Coefficients are the fraction of transmitted inoperability

\[ q(k+1) = A^* q(k) + c^* \]

\[ A^* = \begin{pmatrix} 0 & 0 & 0.3 \\ 0.4 & 0 & 0 \\ 0.2 & 0.6 & 0 \end{pmatrix} ; c^* = \begin{pmatrix} 0 \\ 0 \\ 0.12 \end{pmatrix} \]
IIM example (2)

Infrastructure #1 is affected by a failure of 12%

This induces degradation in #2 and #3

This exacerbates the consequences on # up to 14%

\[
A^d = \begin{bmatrix} 0 & 0 & 0.3 \\ 0.4 & 0 & 0 \\ 0.2 & 0.6 & 0 \end{bmatrix}; c^d = \begin{bmatrix} 0 \\ 0 \\ 0.12 \end{bmatrix}; q(0) = 0
\]
Dependency index & Influence gain

\[
A = \begin{pmatrix}
0 & * & * & * \\
* & 0 & * & * \\
* & * & 0 & * \\
* & * & * & 0 \\
\end{pmatrix}
\]

\[
\rho_j = \sum_i a_{ij}
\]

**dependency index**

Is a measurement of the robustness with respect to the transmitted inoperability

\[
\delta_i = \sum_j a_{ij}
\]

**influence gain**

Is a measurement of the influence that a specific infrastructure has on the global system

Steady-state solution

\[
\bar{x} = (I - A)^{-1} c = S c
\]

If \(A\) is positive and stable, then

\[
S = [I - A]^{-1} = I + A + A^2 + A^3 + \ldots
\]

Overall dependency index and influence gain

\[
\bar{\rho}_j = \frac{1}{n-1} \sum_{i \neq j}^{s_{ij}} \\
\bar{\delta}_i = \frac{1}{n-1} \sum_{j \neq i}^{s_{ij}}
\]

IIM with Technician point of view

Ask to experts the follow question

Which is the impact on your infrastructure of the complete absence of services provided by yyy infrastructure for a time period of zzz

operative technicians’ expertise
(operators’ perceptions)

In this way we try to acquire directly from their expertise an estimation about the dependency parameters to set-up a technical oriented IIM

The scenario

In our case study we consider 11 critical sectors

<table>
<thead>
<tr>
<th>Id</th>
<th>Sector</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Air transportation</td>
</tr>
<tr>
<td>2</td>
<td>Electricity</td>
</tr>
<tr>
<td>3</td>
<td>Wired Telecommunication (TLC wired)</td>
</tr>
<tr>
<td>4</td>
<td>Wireless Telecommunication (TLC wireless)</td>
</tr>
<tr>
<td>5</td>
<td>Water management</td>
</tr>
<tr>
<td>6</td>
<td>Rail transportation</td>
</tr>
<tr>
<td>7</td>
<td>Finance</td>
</tr>
<tr>
<td>8</td>
<td>Naval Ports</td>
</tr>
<tr>
<td>9</td>
<td>Fuel &amp; petroleum grid</td>
</tr>
<tr>
<td>10</td>
<td>Natural Gas</td>
</tr>
<tr>
<td>11</td>
<td>Satellite Communication &amp; Navigation</td>
</tr>
</tbody>
</table>
Time dependencies

We asked to the expert to provide their estimation considering an outage of

a) less than 1 h
b) from 1 to 6 h
c) from 6 to 12 h
d) from 12 to 24 h
e) from 24 to 48 h

To manage the variation of the Leontief coefficient with the outage time, we introduce the «unavailability time»

\[ \tau_i(k + 1) = T_s q_i(k) + \tau_i(k) \]

and, consequently, expand the model

\[
\begin{bmatrix}
q(k + 1) \\
\tau(k + 1)
\end{bmatrix} =
\begin{bmatrix}
A^d(\tau) & 0 \\
T_s I & I
\end{bmatrix}
\begin{bmatrix}
q(k) \\
\tau(k)
\end{bmatrix} +
\begin{bmatrix}
I \\
0
\end{bmatrix} c^d
\]
Coefficient behavior

**Linear + Constant**
The coefficient grows up to a limit

**Single Knee**
Initially a buffer limits the impact. After the buffer is expired the dependency reaches its maximum value.

**Double Knee**
The buffer is used in different moments for instance some basic functions are granted (e.g., cooling for a nuclear power plant)
The curves cross each others, i.e. their relevance/fragility varies with the outage time.
How experts answer

The experts have to use linguistic value extracted from a predefined scale

They have also to express a grade of confidence (accuracy) about each one of their estimation

<table>
<thead>
<tr>
<th>Impact</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>nothing</td>
<td>the event does not induce any effect on the infrastructure</td>
<td>0</td>
</tr>
<tr>
<td>negligible</td>
<td>the event induces some very limited and geographically bounded consequences on services that have no direct impact on the infrastructure’s operativeness</td>
<td>0.05</td>
</tr>
<tr>
<td>very limited</td>
<td>the event induces some geographically bounded consequences on services that have no direct impact on the infrastructure’s operativeness</td>
<td>0.08</td>
</tr>
<tr>
<td>limited</td>
<td>the event induces consequences only on services that have no direct impact on the infrastructure’s operativeness</td>
<td>0.10</td>
</tr>
<tr>
<td>some degradations</td>
<td>the event induces limited and geographically bounded consequences on the capability of the infrastructure to provide its services</td>
<td>0.20</td>
</tr>
<tr>
<td>circumscribed degrada-</td>
<td>the event induces geographically bounded consequences on the capability of the infrastructure to provide its services</td>
<td>0.30</td>
</tr>
<tr>
<td>tion</td>
<td></td>
<td></td>
</tr>
<tr>
<td>significant degradation</td>
<td>the event significantly degrades the capability of the infrastructure to provide its services</td>
<td>0.50</td>
</tr>
<tr>
<td>provided only some</td>
<td>the impact is such that the infrastructure is able to provide national-wide only some essential services</td>
<td>0.70</td>
</tr>
<tr>
<td>services</td>
<td></td>
<td></td>
</tr>
<tr>
<td>quit complete stop</td>
<td>the impact is such that the infrastructure is unable to provide, in some geographically area</td>
<td></td>
</tr>
<tr>
<td></td>
<td>essential services</td>
<td></td>
</tr>
<tr>
<td>stop</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Confidence</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>Good confidence</td>
<td>0</td>
</tr>
<tr>
<td>++</td>
<td>Relative confidence</td>
<td>±0.05</td>
</tr>
<tr>
<td>+++</td>
<td>Limited confidence</td>
<td>±0.10</td>
</tr>
<tr>
<td>++++</td>
<td>Uncertain</td>
<td>±0.15</td>
</tr>
<tr>
<td>+++++</td>
<td>Strongly uncertain</td>
<td>±0.20</td>
</tr>
</tbody>
</table>
To manage the collected information, we adopt Fuzzy Number using a triangular representation.
<table>
<thead>
<tr>
<th>Perceived Severity</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>nothing</td>
<td>the event does not induce any effect on the infrastructure/land</td>
<td>0</td>
</tr>
<tr>
<td>negligible</td>
<td>the event induces some very limited and geographically bounded consequences that have no direct impact on the infrastructure's or land's operativeness</td>
<td>0.025</td>
</tr>
<tr>
<td>very limited</td>
<td>the event induces some geographically bounded consequences that have no direct impact on the infrastructure's or land's operativeness</td>
<td>0.05</td>
</tr>
<tr>
<td>limited</td>
<td>the event induces consequences only on subsystems/zones that have no direct impact on the infrastructure's or land's operativeness</td>
<td>0.1</td>
</tr>
<tr>
<td>circumscribed degradation</td>
<td>the event induces geographically bounded consequences</td>
<td>0.2</td>
</tr>
<tr>
<td>significant degradation</td>
<td>the event significantly degrades the operativeness of the infrastructure/land</td>
<td>0.30</td>
</tr>
<tr>
<td>severe degradation</td>
<td>the impact on the infrastructure/land is severe</td>
<td>0.500</td>
</tr>
<tr>
<td>quite complete stop</td>
<td>the impact is quite catastrophic</td>
<td>0.700</td>
</tr>
<tr>
<td>stop</td>
<td>total disruption</td>
<td>1</td>
</tr>
</tbody>
</table>

**Confidence Scale**

- **Nothing** (Certain)
- **Limited** (Relative Confidence)
- **Circumscribed** (Excellent Confidence)
- **Significant** (Relative Confidence)
- **Quite Catastrophic** (Excellent Confidence)

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**Criticality Scale**
IIM Fuzzy System (difference inclusion system)

\[ x(k + 1) = Hx(k), \quad x(0) = X_0 \in E^n \]

\( H \) is a \( nxn \) fuzzy-value matrix, i.e. \( h_{ij} \in \mathbb{E} \)

To solve, we have to translate the fuzzy-equation into a family of discrete difference inclusions

\[ [x]^{\alpha}(k + 1) \in [H]^{\alpha}[x]^{\alpha}(k); \]

Results IIM Fuzzy

Consequences of a «severe failure» in the electric grid ($c_2=[0.5, 0.6, 0.7]$) in conjunction with a «moderate failure» in the wired TLC network ($c_3=[0.2, 0.3, 0.35]$)

Overall dependency index and influence gain
To identify most critical infrastructures (e.g. those on which prioritise the resource)

Topological approaches
Coupling of heterogeneous networks

Flow model peculiar for each type of network

**Electric Grid - DC Power Flow**

\[
F_{km} = \frac{(\theta_k - \theta_m)}{x_{km}}
\]

\[
P_k = \sum_m F_{km} = \theta_k \sum_m x_{km}^{-1} - \sum_m \frac{\theta_m}{x_{km}}
\]

\[
P = B \theta
\]

\[
\text{con } B_{km} = -\frac{1}{x_{km}} \quad \text{e } B_{kk} = \sum_l \frac{1}{x_{kl}}
\]

\[
\sum_i P_i = 0
\]

With constraints on maxima power flow on each link and maxima angle shift

- \( \theta_{km} < 30 \text{ degree} \)
- \( |F_{km}| < F_{km}^{\text{max}} \)

The network is perturbed eliminating 1 or more links

The solution is calculating considering also a re-

\[
\Delta P = \sum_{i \in \text{loads}} \left[ P_i - P_i^0 \right]
\]
TLC model

- The network delivery paketes
- At each time instant, each node generates a packet, with probability $\lambda$, addressed to a random destination node
- The packet flow via adjacent nodes following static routing table via random route

$$P(j_1) = \frac{e^{-\beta X_1}}{e^{-\beta X_1} + e^{-\beta X_2}}$$
$$P(j_2) = \frac{e^{-\beta X_2}}{e^{-\beta X_1} + e^{-\beta X_2}}$$

- Any node can manage just a packet each time and all the nodes have the same throughput
Complex network analysis

\[ GARR \text{ node } k = \begin{cases} \text{on} & \text{if } P_i \geq \alpha P_i^{(0)} \\ \text{off} & \text{otherwise} \end{cases} \]

TLC delivery time

\[ \text{QoS}_{\text{TLC}} = \frac{\frac{m}{M}}{\frac{\langle T \rangle}{\langle T_0 \rangle}} \]

\[ \text{QoS}_{\text{El.}} = 1 - \frac{\Delta P_i}{\Sigma P_i^0} \]
Cyber-Physical Systems

Malicious manipulations/faults may happen at all levels
Distributed detection

A single node, exploiting only local information able to communicate only with its neighbors is able to detect (and restore) faults in the network?

Specifically we want to detect:

• Node(s) fault (how any all over the network)
• Link(s) fault (how many all over the network)
• Presence of cycle (and resolve them by links swapping)
• System controllability (and restore it by link swapping)
Distributed detection

We develop an distributed approach where each node performs a set of max-, min-, and average consensus to locally calculate the information and to perform the links’ swap needed to restore the nominal condition.

The Consensus problem is to manipulate the inputs for ith node provided only by its neighbors.

\[ x_i(t + 1) = U_i(\{x_j(t) : v_j \in N_i^{in} \cup \{i\}\}) \]

In order to guarantee that ALL the nodes converges to a given function of the initial condition stored independently by each single node.

\[ \lim_{t \to \infty} x_i(t) = \chi(x_1(0), \ldots, x_n(0)) \quad \forall i = 1, \ldots, n. \]
We develop a set of algorithm that in at least $n$ steps is able to detect any faults, detecting also the presence of cycle (and restoring it) so as if the system is controllable (and eventually restore it)

**Algorithm 1: Distributed Node and Edge Variation Detection**

```plaintext
/* Initialization */
max-id(0) = max-consensus(id_i, G_c(0));
is-leader_i(0) = 0;
if max-id(0) == id_i then
    is-leader_i(0) = 1;
end if

\[
n_i(0) = \frac{1}{\text{average-consensus}(\text{is-leader}_i(0), G_c(0))};
\]

\[
e_i(0) = \text{average-consensus}(d_i(0)/2, G_c) n_i(0);
\]

/* Check if something changes in $t = T$ */
if $t == T$ then
    max-id(T) = max-consensus(id_i, G_c(T));
is-leader_i(T) = 0;
if max-id(T) == id_i then
    is-leader_i(T) = 1;
end if

\[
n_i(T) = \frac{1}{\text{average-consensus}(\text{is-leader}_i(T), G_c(T))};
\]

/* I) Variation of the number of nodes*/
if $n_i(T) \neq n_i(0)$ then
    Detect variation of the number of nodes;
end if

/* II) Variation of the number of edges*/
e_i(T) = \text{average-consensus}(d_i(T)/2, G_c) n_i(T);
if $e_i(T) \neq e_i(0)$ then
    Detect variation of the number of edges;
end if
```

end if
Case study: IEEE118 Bus Case

Faulted Network (2 generators and 7 links)

The system is no more controllable and there are some cycles

Distributed detection

#2 nodes loss

#7 links loss
Aciclycity & Controllability Restored

The swaps occur all over the network
The FACIES European Project

online identification of Failure and Attack on interdependent Critical InfrastructurES

http://facies.dia.uniroma3.it/

With the financial support of the “Prevention, Preparedness and Consequence Management of Terrorism and other Security-related Risks Programme

http://facies.dia.uniroma3.it/
How identify failures/attacks having a partial and limited vision of the process in the presence of interdependencies and taking into account also the possible cyber-data manipulation and cyber failure?
FACIES Architecture

- Sensors
- Pumps
- Valves
- PLC
- SCADA (iFix)
- HMI
- SWITCH
- IDS EXPERT SYSTEM
- RISK PREDICTOR
- FAULT DETECTION
Physical, Cyber, Logic and Geographic dependencies
Critical Infrastructure Preparedness and Resilience Research (NoE)

Network of Excellence, co-funded by FP7
Term: 1.3.2013–28.2.2017

Partners

1. Coordinator: Fraunhofer IAIS, DE
2. ENEA, IT
3. TNO, NL
4. UIC, FR
5. CEA, FR
6. EC Joint Research Centre, EU
7. Deltares, NL
8. University of Cyprus, CY
9. University of Technology and Life Sciences, PL
10. Università UCBM, IT
11. University of British Columbia, CA
12. ACRIS GmbH, CH
(Real-time) hazmat analysis

Direct impact analysis

First order impact analysis

Higher order impact analysis
Each geographical point is associated with a “dynamical” Threat Strength Matrix, containing the predicted occurrence and the strength expected for all the given perturbation.

<table>
<thead>
<tr>
<th>Lat.xx</th>
<th>Long.yy</th>
<th>Vulnerability grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>Threat name</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Earthquake (ground acceleration)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Strong Wind</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>Lightening</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Heavy snowfall</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Ice</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Landslide</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Flash flood</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Flooding</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Mud flows</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Debris avalanches</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Heavy Rain</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Strom surge</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Each CI element is characterised by a Vulnerability Matrix which indicates the maximum perturbation strength (originating from each considered hazard) that it could sustain before its physical failure.
Overlaying the **Vulnerability and the Threat Strength matrices** will allow to predict the level of damage that the predicted threat(s) will produce on each CI element present in the different areas under the DSS control.

Reported data are obtained through the collaboration with project RoMA.
January 31, 2014
Thank You!

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